Prove that a finite set of points cannot have any accumulation point.

Solution

Let $S = \{z_1, z_2, \dots, z_n\}$.
Let z be some arbitrary point (may or may not be contained in S), and let

$$d = \inf\{|z - z_i| : z_i \in S\}.$$

This establishes that there is a minimum distance between any arbitrary point z and a nearby point $z_i : z_i \in S$.
Now we can let $B_\varepsilon(z)$ be a neighborhood around the arbitrary point z. If the set S were infinite then there would always be some radius ε in which a point in $S \setminus \{z\}$ is included, therefore proving there is an accumulation point. However because S is finite, and ε can be provided such that $\varepsilon < d$. In other words, if ε is less than the minimum distance between some arbitrary point z and a point z_i then

$$\forall x \in B_\varepsilon(z), x \notin S.$$

Therefore there is no accumulation point.