Question

(1+x+x^2+......)(1+x^2+x^4+.......)(1+x^3+x^6......) in this case what is the coefficient of x^6 ?

Solution

Writing all summands explicitly up to the 6th power we rewrite this expression

$$w = (1 + x + x^2 + \cdots)(1 + x^2 + x^4 + \cdots)(1 + x^3 + x^6 + \cdots) =$$

$$= (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 \dots)(1 + x^2 + x^4 + x^6 \dots)(1 + x^3 + x^6 + \cdots)$$

The product of the two last multipliers is equal to

$$(1 + x^2 + x^4 + x^6 \dots)(1 + x^3 + x^6 + \cdots) = (1 + x^2 + x^3 + x^4 + x^5 + 2x^6 + \cdots)$$

The substitution of this product in the previous expression, multiplication of the two multipliers and the cancellation give the sought coefficient

$$w = (1 + x + x^2 + x^3 + x^4 + x^5 + x^6 \dots)(1 + x^2 + x^3 + x^4 + x^5 + 2x^6 \dots) =$$

$$= 1 + x + A_2x^2 + A_3x^3 + A_4x^4 + A_5x^5 + 7x^6 \dots$$

where A_2, A_3, A_4, A_5 are some numerical coefficients.

Thus the coefficient of x^6 is equal to 7.

Answer:

It is 7.