

Question: The percentage of American men who say they would marry the same woman if they had to do it over again is 65% what is the probability that in a group of 10 married American men, no more than 3 will claim that they would marry the same woman again? what is a probability that at least 6 will say this?

Solution: Event A: "randomly chosen men belongs to a group of men that would marry the same woman if they had to do it over again". $P(A) = 0.65$.

Let X be a random variable of the number of successes in a sequence of 10 independent experiments, where success probability is $P(A) = 0.65$. X follows the binomial distribution with parameters $n = 10$, $p = 0.65$. $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k} = \binom{10}{k} 0.65^k 0.35^{10-k}$ for $k = 0, 1, \dots, 10$.

- The probability that in a group of 10 married American men, no more than 3 will claim that they would marry the same woman again is

$$\begin{aligned} P(X \leq 3) &= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) = \binom{10}{0} 0.65^0 0.35^{10} + \binom{10}{1} 0.65^1 0.35^9 + \\ &\quad \binom{10}{2} 0.65^2 0.35^8 + \binom{10}{3} 0.65^3 0.35^7 = 0.35^7 (0.35^3 + 10 * 0.65 * 0.35^2 + 45 * 0.65^2 * \\ &\quad 0.35 + 120 * 0.65^3) = 0.35^7 * 40.4485 \approx 0.026. \end{aligned}$$

- The probability that at least 6 will say this:

$$\begin{aligned} P(X \geq 6) &= 1 - P(X \leq 5) = 1 - (P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)) = 1 - \\ &\quad (0.35^7 * 40.4485 + P(X = 4) + P(X = 5)) = 1 - 0.35^7 * 40.4485 - \binom{10}{4} 0.65^4 0.35^6 - \\ &\quad \binom{10}{5} 0.65^5 0.35^5 = 1 - 0.35^7 * 40.4485 - 0.65^4 0.35^4 (210 * 0.35^2 + 252 * 0.65 * 0.35) = \\ &\quad 1 - 0.35^7 * 40.4485 - 0.65^4 0.35^4 * 83.055 \approx 0.751. \end{aligned}$$

Answer: The probability that in a group of 10 married American men, no more than 3 will claim that they would marry the same woman again is ≈ 0.026 .

The probability that at least 6 will say this is ≈ 0.751 .