

Condition

$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+16}-4}{x}$

Solution

$$\lim_{x \rightarrow 0} \frac{\sqrt{x^2+16}-4}{x} = \frac{\sqrt{0+16}-4}{0} = \frac{4-4}{0} = \left| \frac{0}{0} \right|$$

Multiplying both the numerator and denominator by $\sqrt{x^2+16}+4$:

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{(\sqrt{x^2+16}-4) \cdot (\sqrt{x^2+16}+4)}{x \cdot (\sqrt{x^2+16}+4)} &= \lim_{x \rightarrow 0} \frac{(\sqrt{x^2+16})^2 - 4^2}{x \cdot (\sqrt{x^2+16}+4)} = \\ &= \lim_{x \rightarrow 0} \frac{x^2+16-16}{x \cdot (\sqrt{x^2+16}+4)} = \lim_{x \rightarrow 0} \frac{x^2}{x \cdot (\sqrt{x^2+16}+4)} = \lim_{x \rightarrow 0} \frac{x}{\sqrt{x^2+16}+4} = \frac{0}{\sqrt{0+16}+4} = \frac{0}{4+4} = 0. \end{aligned}$$

Answer

Bonus

If the condition was slightly different

Condition

$\lim_{x \rightarrow 0} \sqrt{x^2 + 16} - \frac{4}{x}$

Solution

$$\lim_{x \rightarrow 0} \left(\sqrt{x^2 + 16} - \frac{4}{x} \right) = \sqrt{0 + 16} - \frac{4}{0} = -\infty.$$

Answer

$-\infty$.