Answer on question 36292 – Math – Calculus

$$f(x) = \frac{x^2 + 7}{x - 3}$$

Find the vertical and horizontal/slant asymptotes.

Solution

The line *x* = *a* is a *vertical asymptote* of the graph of the function *y* = *f*(*x*) if at least one of the following statements is true:

1. $\lim_{x \to a^-} f(x) = \pm\infty$
2. $\lim_{x \to a^+} f(x) = \pm\infty.$

Let us find the domain of our function. This is the fraction that is why the denominator isn't equal to 0: $x - 3 \neq 0$ or $x \neq 3$. This is the suspicious point. Consider the following limits

$$\lim_{x \to 3-} \frac{x^2 + 7}{x - 3} = \lim_{x \to 3-} \frac{x^2 - 9 + 9 + 7}{x - 3} = \lim_{x \to 3-} \frac{(x - 3)(x + 3) + 16}{x - 3} = \lim_{x \to 3-} \left(x + 3 + \frac{16}{x - 3}\right) = -\infty,$$

$$\lim_{x \to 3+} \frac{x^2 + 7}{x - 3} = \lim_{x \to 3+} \left(x + 3 + \frac{16}{x - 3}\right) = +\infty.$$

We obtain that the x=3 is a vertical asymptote.

Horizontal asymptotes are horizontal lines that the graph of the function approaches as *x* → ±∞. The horizontal line *y* = *c* is a horizontal asymptote of the function *y* = *f*(*x*) if

$\lim_{x \to -\infty} f(x) = c$ or $\lim_{x \to +\infty} f(x) = c$.

Consider following limits

$$\lim_{x \to -\infty} \frac{x^2 + 7}{x - 3} = \lim_{x \to -\infty} \left(x + 3 + \frac{16}{x - 3}\right) = -\infty;$$

$$\lim_{x \to \infty} \frac{x^2 + 7}{x - 3} = \lim_{x \to \infty} \left(x + 3 + \frac{16}{x - 3}\right) = \infty.$$

This function has no horizontal asymptotes.

To find the oblique asymptote we first need to find the following limits

$$m = \lim_{x \to \pm\infty} \frac{x^2 + 7}{x(x - 3)} = \lim_{x \to \pm\infty} \frac{x^2 + 7}{x^2 - 3x} = 1;$$

$$n = \lim_{x \to \pm\infty} \left(\frac{x^2 + 7}{x - 3} - mx\right) = \lim_{x \to \pm\infty} \left(\frac{x^2 + 7}{x - 3} - \frac{x(x - 3)}{x - 3}\right)$$
$$= \lim_{x \to \pm\infty} \left(\frac{x^2 + 7 - x^2 + 3x}{x - 3}\right) = \lim_{x \to \pm\infty} \left(\frac{7 + 3x}{x - 3}\right) = 3.$$

Therefore, the oblique asymptote is $y = mx + n = x + 3$.

Answer: x=3 and y=x+3.