Probability of India winning match against Pakistan is 0.5 find the probability that India's 2nd win occur s in 3rd test in a 5 match series?

Solution

The probability of India winning a test match against Pakistan $=\frac{1}{2}$.

=> The probability of India losing a test match against Pakistan = $1 - \frac{1}{2} = \frac{1}{2}$.

The criteria here is "India's second win should occur at the third test".

The following table summarizes all the possibilities:

	1 st	2^{nd}	3 rd	$P(E) = P(E_1) * P(E_2)$
	Match	Match	Match	$*P(E_3)$
India loosing all of the first	L	L	L	1 1 1 1
three				$\frac{1}{2} * \frac{1}{2} * \frac{1}{2} = \frac{1}{8}$
India winning one of the first	W	L	L	1 1 1 1
three				$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
	L	W	L	1 1 1 1
				$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
	L	L	W	1 1 1 1
				$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$
India winning two of the first	L	W	W	1 1 1 1
three				$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$
	W	L	W	1 1 1 1
				$\overline{2}^*\overline{2}^*\overline{2}^=\overline{8}$
	W	W	L	1 1 1 1
				$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$
India winning all of the first	W	W	W	1 1 1 1
three				$\overline{2} * \overline{2} * \overline{2} = \overline{8}$

Out of all the 8 possibilities, the desired possibilities are two only (shown bold italic)

Probability that India's 2nd win occur s in 3rd test in a 5 match series $P = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$.

Answer: $\frac{1}{4}$.