the displacement of a boat relative to water is represented by (3t)i+4(t2-1)j(read as 4 (t square-1)j) and that of water relative to ground is i-(et)j (read as i-(e power t)j).what is the velocity of the boat relative to ground if i&j represent 1km/hour east and north respectively?

Solution

The displacement of a boat relative to water $\overrightarrow{d_1}$ is represented by

$$\overrightarrow{d_1} = (3\mathrm{t})\mathrm{i} + 4(\mathrm{t}^2 - 1)\mathrm{j}.$$

The displacement of water relative to ground $\overrightarrow{d_2}$ is represented by

$$\overrightarrow{d_2} = \mathrm{i} - (\mathrm{e}^{\mathrm{t}})\mathrm{j}.$$

The displacement of a boat relative to ground $\overrightarrow{d_3}$ is the sum of displacements of a boat relative to water $\overrightarrow{d_1}$ and that of water relative to ground $\overrightarrow{d_2}$:

$$\overrightarrow{d_3} = \overrightarrow{d_1} + \overrightarrow{d_2} = \big((3\mathrm{t})\mathrm{i} + 4(\mathrm{t}^2 - 1)\mathrm{j}\big) + (\mathrm{i} - (\mathrm{e}^{\mathrm{t}})\mathrm{j}) = (3t + 1)\mathrm{i} + (4(\mathrm{t}^2 - 1) - \mathrm{e}^{\mathrm{t}})\mathrm{j}.$$

The velocity of the boat relative to ground

$$\vec{v} = \frac{d}{dt}\overrightarrow{d_3} = i\frac{d}{dt}(3t + 1) + j\frac{d}{dt}(4(\mathrm{t}^2 - 1) - \mathrm{e}^{\mathrm{t}}).$$

Let's find components of the velocity due east and north:

$$v_{east} = \frac{d}{dt}(3t + 1) = 3\frac{\mathrm{km}}{\mathrm{h}};$$

$$v_{north} = \frac{d}{dt}(4(\mathrm{t}^2 - 1) - \mathrm{e}^{\mathrm{t}}) = (8t - e^t)\frac{\mathrm{km}}{\mathrm{h}}.$$

Answer: $3\frac{\mathrm{km}}{\mathrm{h}}$ east and $(8t - e^t)\frac{\mathrm{km}}{\mathrm{h}}$ north.