Please show that the vector a is orthogonal to the hyperplane H = H(a,€); that is, if u and v are in H, then a is orthogonal to u - v.

Solution.

We present an example to illustrate this statement.

If the vector $\vec{a}$ is orthogonal to the H, then $\vec{a}$ is a normal vector for H. Let $\vec{a} = (2,3)$ and H is a straight line with equation

$$2x + 3y = 1$$

Find points on H. Suppose point $(x; y)$ lies on H. We note that when $x = 2$, $y = -1$, so $\vec{u} = (2, -1)$ lies on H. Thus,

$$(2,3) \cdot \big((x, y) - (2, -1)\big) = 0,$$

or, equivalently,

$$(2,3) \cdot (x - 2, y + 1) = 0,$$

is a normal equation for H. Since $\vec{v} = (-1,1)$ also lies on H, one of directions of the straight line $H\ is\ \vec{v} - \vec{u} = (-3,2)$.

Note that

$$\vec{a} \cdot (\vec{v} - \vec{u}) = (2,3) \cdot (-3,2) = 0,$$

so $\vec{a}$ is orthogonal to $\vec{v} - \vec{u}$.