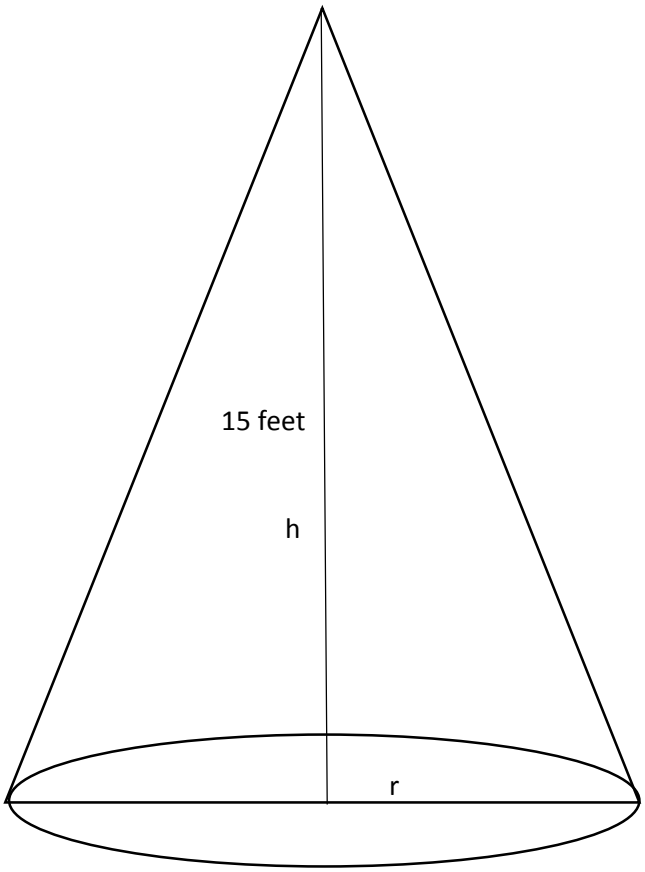
Answer on question 36049 – Math – Calculus

Sand is falling into a cone at a rate of 10 cubic feet per minute. The diameter of the base of the cone is approximately 3 times the altitude. At what rate is the height of the pile changing when the pile is 15 feet high?

Answer:



The radius of the base is h/6 feet. Then the cone volume is

$$V_0 = \frac{1}{3}\pi r^2 h = \frac{\pi h^3}{108} = \frac{125\pi}{4}.$$

The volume of free space is decrease with a rate of 10 cubic feet per minute. So the free volume at time t is

$$V_t = V_0 - 10t = \frac{125\pi}{4} - 10t.$$

On the other hand this volume equals $\frac{\pi h(t)^3}{108}$, where $h(t)$ is the height of free cone, and we get

$$\frac{\pi h(t)^3}{108} = \frac{125\pi}{4} - 10t$$

Or

$$h(t) = \sqrt[3]{\frac{125\pi - 40t}{\pi}}$$

the rate of decreasing this height is

$$h'(t) = -\frac{40}{\sqrt[3]{\pi(125\pi - 40t)^2}}.$$

The rate of increasing the height of sand is

$$\frac{40}{\sqrt[3]{\pi(125\pi - 40t)^2}}$$

Answer: $\frac{40}{\sqrt[3]{\pi(125\pi-40t)^2}}$.