Kayla:

A recent survey determined the amount of time that 30 customers spent in a Stop-n-Shop convenience store. The data were normally distributed, with a mean of 12 minutes and a standard deviation of 2 minutes.

Based on the results, if 500 customers come to the stop-n-shop about how many will spend between 10 and 14 minutes in the store?

MULTIPLE CHOICE:

A: 170
B: 200 (this was my answer, it is incorrect)
C: 250
D: 340

Solution.

If the data were normally distributed, then a density function of normal distribution is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

We have a standard deviation $\sigma = 2\ \mathrm{min}$, a mean $\mu = 12\ \mathrm{min}$.

So

$$f(x) = \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-12)^2}{8}}$$

We must find how many customers will spend between 10 and 14 minutes, so let's find a probability using the normal distribution:

$$P = \int_{10}^{14} f(x)dx = \int_{10}^{14} \frac{1}{2\sqrt{2\pi}} e^{-\frac{(x-12)^2}{8}} dx = \frac{1}{2\sqrt{2\pi}} \int_{10}^{14} e^{-\frac{(x-12)^2}{8}} dx$$

We know that

$$\int_a^b e^{-t^2} dt = \frac{\sqrt{\pi}}{2} (\operatorname{erf} b - \operatorname{erf} a),$$

where $\operatorname{erf} t$ is the error function.

$$\frac{(x-12)^2}{8} = \left(\frac{x-12}{2\sqrt{2}}\right)^2$$

$$\frac{x-12}{2\sqrt{2}} = t$$

$$x = 2\sqrt{2}t + 12$$

$$dt = \frac{1}{2\sqrt{2}} dx$$

So

$$\frac{1}{2\sqrt{2\pi}}\int_{10}^{14} e^{-\frac{(x-12)^2}{8}}dx = \left|\frac{(x-12)^2}{8} = \left(\frac{x-12}{2\sqrt{2}}\right)^2 = t^2\right| = \frac{2\sqrt{2}}{2\sqrt{2\pi}}\int_{-1/\sqrt{2}}^{1/\sqrt{2}} e^{-t^2}dt$$
$$= \frac{1}{\sqrt{\pi}}\int_{-1/\sqrt{2}}^{1/\sqrt{2}} e^{-t^2}dt = \frac{\sqrt{\pi}}{\sqrt{\pi}}\operatorname{erf}\frac{1}{\sqrt{2}} \approx 0.68$$

Then number of the customers that will spend between 10 and 14:

$$N = 500 \cdot I \approx 500 \cdot 0.68 = 340$$

Answer: D. 340.