

The Senate consists of 100 senators, 2 from each of the 50 states. A Senate committee of six is to be formed.

(a) How many ways are there to form the committee from the 100 senators?

(b) If at most one senator from each state can serve on the committee, how many ways are there to form the committee?

Solution

(a) Number of ways to choose 6 senators from total 100 senators

$$N_{100,6} = \binom{100}{6} = \frac{100 * 99 * 98 * 97 * 96 * 95}{6 * 5 * 4 * 3 * 2 * 1} = 1192052400.$$

(b) There are 50 states and 2 senators from each state, total 100. According to question, from each state at most 1 senator can be selected.

So, firstly we choose 6 states from which we have to select senators.

Number of ways to choose 6 states from total 50 states

$$N_{50,6} = \binom{50}{6} = \frac{50*49*48*47*46*45}{6*5*4*3*2*1} = 15890700.$$

Now, from each state we can choose senator in 2 ways.

Total number of ways to choose 6 senator from given 6 particular states

$$N_{6,6} = 2^6 = 64.$$

So, total number of ways to select 6 senators

$$N_{\text{tot}} = N_{50,6} * N_{6,6} = 15890700 * 64 = 1017004800.$$

Answer: (a) 1192052400; (b) 1017004800.