Answer on question 35970 – Math – Multivariable calculus

Given the function g(t1,t2) = 2/(t1t2) + t1t2 + t1. Show that there is no solution to the problem of minimizing this function over t1>0, t2>0. Can g(t1,t2) ever be smaller than $2\sqrt{2}$?

Solution

$$\begin{cases} \mathrm{g}(\mathrm{t}_1, \mathrm{t}_2) = \dfrac{2}{\mathrm{t}_1\mathrm{t}_2} + \mathrm{t}_1\mathrm{t}_2 + \mathrm{t}_1 \to \min \\ t_1 > 0 \\ t_2 > 0 \end{cases}$$

According to the necessary condition for an extrema of function of two variables we need to find the solution of the following system

$$\begin{cases} \dfrac{\partial g(\mathrm{t}_1, \mathrm{t}_2)}{\partial t_1} = 0 \\ \dfrac{\partial g(\mathrm{t}_1, \mathrm{t}_2)}{\partial t_2} = 0 \end{cases} \qquad \begin{cases} -\dfrac{2}{t_1^2 t_2} + t_2 + 1 = 0 \\ -\dfrac{2}{t_1 t_2^2} + t_1 = 0 \end{cases} \qquad (*)$$

From the second equation we get

$$t_1^2 = \frac{2}{t_2^2}$$

Substitute this into the first equation

$$-\frac{2t_2^2}{2t_2} + t_2 + 1 = 0, \qquad 1 = 0.$$

This equation has no solutions.

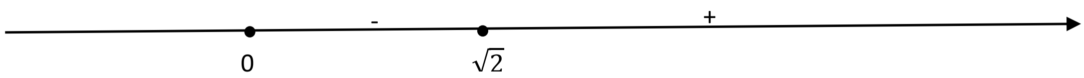
Therefore, the system of equations has no solution and the function $g(\mathrm{t}_1, \mathrm{t}_2)$ has no extrema.

Now let us consider the function $f(t_1, t_2) = \frac{2}{\mathrm{t}_1\mathrm{t}_2} + \mathrm{t}_1\mathrm{t}_2$, where $t_1 > 0, t_2 > 0$. Obviously that $f(t_1, t_2) < g(t_1, t_2)$.

Find the minimum of the function $f(t_1, t_2) = \frac{2}{\mathrm{t}_1\mathrm{t}_2} + \mathrm{t}_1\mathrm{t}_2$. Let $t_1 t_2 = x$ then $f(x) = \frac{2}{x} + x$.

$$f'(x) = -\frac{2}{x^2} + 1 = \frac{x^2 - 2}{x^2} = 0$$

$$x = \sqrt{2}.$$



At the point $\sqrt{2}$ this function has a minimum.

$$f(\sqrt{2}) = \frac{2}{\sqrt{2}} + \sqrt{2} = \frac{4}{\sqrt{2}} = 2\sqrt{2}.$$

And we get $2\sqrt{2} \le f(t_1, t_2) < g(t_1, t_2)$.

Answer: the function $g(t_1, t_2)$ has no extrema and never be smaller than $2\sqrt{2}$.