Find the area of triangle whose perimeter is 48 cm and length of altitudes from opposite vertex are 20 cm, 28 cm and 35 cm respectively.

Solution:

The formula of the area **T** of a triangle is:

$$T = ½ * a*h_a;$$

where **a** is the length of the base of the triangle, and $\mathbf{h_a}$ is the height of the triangle conducted to side **a**.

Also we can write this formula for every side of triangle:

$$T = ½ * b*h_b;$$

where **b** is the length of the base of the triangle, and $\mathbf{h_b}$ is the height of the triangle conducted to side **b.**

$$T = ½ * c*h_c;$$

where **c** is the length of the base of the triangle, and $\mathbf{h_c}$ is the height of the triangle conducted to side **c.**

As you understand we can equate all this equation and we will get

$$T = ½ * a*h_a = ½ * b*h_b = ½ * c*h_c$$

Then

$$a*h_a = b*h_b = c*h_c$$

The perimeter **P** of a triangle is:

$$P = a + b+ c$$

In our case h_a = 20 cm, h_b = 28 cm, h_c = 35 cm and P = 48 cm;

Then

$$a + b+ c = 48$$

$$20*a = 28*b = 35*c$$

Now we are solving this problem

a = 35*c/20

b = 35*c/28

35*c/20 + 35*c/28 + c = 48

7*c/4 + 5*c/4 + c = 48

$7*c + 5*c + 4*c = 192$

$16*c = 192$

$c = 12$

Then of the area **T** of a triangle is:

$$T = \frac{1}{2} * 12*35 = 210$$

Answer: the area of the triangle is **210 cm²**