Question: I put 32 chess pieces in a hat. I then proceeded to pull them out 2 at a time. If they are both black I put them in one pile. If they are both white, I put them in a second pile. If there is one of each color, I put them in a third pile. What is the probability that the number of pieces in the all black pile and the all white pile are equal?

Solution: Denote the number of white pieces in the hat at the beginning as w. Then the number of black pieces is $b = 32 - w$.

Let $2k$ be a number of chess pieces in all white pile, $2m$ – in all black pile and $2n$ – in the third pile. First pile contains $2k$ white pieces, second – $2m$ black pieces and third – n black and n white pieces.

Therefore, the total number of white pieces in all piles is $2k+n$, which is equal to w (the number of white pieces in the hat at the beginning). $2k+n = w$. Similarly, we obtain, that $2m+n = b$.

Subtracting the second equality from the third, we obtain that: $2k-2m = w - b$, and $2k = 2m + (w - b)$. So, the number of pieces in the all black pile and the all white pile are equal if and only if $w = b$. Therefore, the sought probability is 1, when $w = b = 16$, and 0, if $w \neq b$.

Answer: 1, if the number of white pieces and the number of black pieces in the hat at the beginning are equal, and 0, if not.