We have the matrix:

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

Let's find eigenvalues:

$$\det(A - \lambda I) = \det\begin{pmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 2 \\ 2 & 1 & 2-\lambda \end{pmatrix}$$
$$= (2-\lambda)\big((3-\lambda)(2-\lambda)-2\big) - (2(2-\lambda)-4) + \big(2-2(3-\lambda)\big)$$
$$= -\lambda^3 + 7\lambda^2 - 10\lambda + 4 = -(\lambda-1)(\lambda^2 - 6\lambda + 4) = 0$$

The roots of this polynomial are:

$$\lambda_1 = 1$$

$$\lambda_{2,3} = 3 \pm \sqrt{5}$$

Let's find eigenvectors corresponding to each of the eigenvalues.

Eigenvectors for given λ are the fundamental solutions of the system

$$(A - \lambda I)x = 0$$

For $\lambda_1 = 1$ we have such matrix:

$$A - \lambda I = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 2 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$$

Eigenvector corresponding to λ_1 is thus $v_1 = C_1(0,-1,1)$, where C_1 is a real constant, $C_1 \neq 0$.

For $\lambda_2 = 3 - \sqrt{5}$:

$$A - \lambda I = \begin{pmatrix} \sqrt{5}-1 & 1 & 1 \\ 2 & \sqrt{5} & 2 \\ 2 & 1 & \sqrt{5}-1 \end{pmatrix} \sim \begin{pmatrix} \sqrt{5}-3 & 0 & 2-\sqrt{5} \\ 2-2\sqrt{5} & 0 & -3+\sqrt{5} \\ 2 & 1 & \sqrt{5}+1 \end{pmatrix} \sim \begin{pmatrix} \sqrt{5}-3 & 0 & 2-\sqrt{5} \\ 2 & 1 & \sqrt{5}+1 \end{pmatrix}$$

Corresponding eigenvector equals to

$v_2 = C_2\big(1-\sqrt{5}, 2-2\sqrt{5}, 4\big)$, where C_2 is a real constant, $C_2 \neq 0$.

For $\lambda_3 = 3 + \sqrt{5}$:

$$A - \lambda I = \begin{pmatrix} -\sqrt{5}-1 & 1 & 1 \\ 2 & -\sqrt{5} & 2 \\ 2 & 1 & -\sqrt{5}-1 \end{pmatrix} \sim \begin{pmatrix} -\sqrt{5}-3 & 0 & 2+\sqrt{5} \\ 2+2\sqrt{5} & 0 & -3-\sqrt{5} \\ 2 & 1 & \sqrt{5}+1 \end{pmatrix} \sim \begin{pmatrix} -\sqrt{5}-3 & 0 & 2+\sqrt{5} \\ 2 & 1 & \sqrt{5}+1 \end{pmatrix}$$

Corresponding eigenvector equals to

$v_3 = C_3(1+\sqrt{5}, 2+2\sqrt{5}, 4)$, where C_3 is a real constant, $C_3 \neq 0$.