Question: Suppose that three missiles are available to fire at three targets, called Target 1, Target 2, and Target 3. Each time a missile is fired at Target 1 it will hit it with probability 0.9. Each time a missile is fired at Target 2 it will hit it with probability 0.8. Each time a missile is fired at Target 3 it will hit it with probability 0.7. Missiles will be fired, one at a time, at Target 1 until it is hit. If Target 1 is hit, any remaining missiles will be fired at Target 2 until it is hit, and if Target 2 is hit before the last of the three missiles is fired, the last missile will be fired at Target 3. Letting H1 be the event that the first missile fired hits a target, and H2 be the event that the second missile fired hits a target, explain why H1 and H2 are not independent events.

Answer: Two events *A* and *B* are independent if and only if their joint probability equals the product of their probabilities: $P(A\cap B)=P(A)P(B)$. So we should prove that $P(H_1\cap H_2)\neq P(H_1)\,P(H_2)$.
Let T(i, j) be the event that a missile number j, fired at the Target i, hits it, i=1, 2, 3, j=1, 2, 3. T(i, j) are all independent events. P(T(1,j))=0.9, P(T(2,j))=0.8, P(T(3,j))=0.7, j=1, 2, 3.

a) $H_1\cap H_2$ is the event that the first missile hits Target 1, and then second missile hits Target 2. $P(H_1\cap H_2)=P(T(1,1)\cap T(2,2))=P(T(1,1))P(T(2,2)) = 0.9*0.8 = 0.72$.
b) H_1 is the event that the first missile hits a Target 1. $P(H_1)=P(T(1,1))=0.9$.
c) H_2 is the event that the second missile hits a target. There two possible cases:
   1) First missile hits a Target 1 and then second hits Target 2 – $T(1,1)\cap T(2,2)$.
   2) First missile misses a Target 1 and then second hits Target 1 – $\overline{T(1,1)}\cap T(1,2)$.

   $H_2 = \left(T(1,1)\cap T(2,2)\right)\cup\left(\overline{T(1,1)}\cap T(1,2)\right)$, and $\left(T(1,1)\cap T(2,2)\right)\cap\left(\overline{T(1,1)}\cap T(1,2)\right)=\emptyset$.
   Therefore, $P(H_2) = P(T(1,1)\cap T(2,2)) + P(\overline{T(1,1)}\cap T(1,2))=P(T(1,1))P(T(2,2)) + (1 - P(T(1,1)))P(T(1,2)) = 0.9*0.8 + (1 - 0.9)*0.9 = 0.81$.

Obtained, that $P(H_1)\,P(H_2)=0.9*0.81= 0.729$, which is not equal to $P(H_1\cap H_2) = 0.72$. Therefore, H_1 and H_2 are not independent events■