Question #35544, Math,Calculus

Compute $\frac{\partial}{\partial\alpha}\frac{\alpha^2+\beta^2}{1+e^{\alpha\beta}}$

Solution.

In computing partial derivatives we can use all the rules for ordinary derivatives.

Using the quotient rule

$$\frac{\partial}{\partial\alpha}\frac{u}{v}=\frac{\frac{\partial u}{\partial\alpha}v-u\frac{\partial v}{\partial\alpha}}{v^2},$$

where $u=\alpha^2+\beta^2, v=1+e^{\alpha\beta}$ and treating β like a constant, we receive

$$\frac{\partial}{\partial\alpha}\frac{\alpha^2+\beta^2}{1+e^{\alpha\beta}}=\frac{\frac{\partial}{\partial\alpha}(\alpha^2+\beta^2)(1+e^{\alpha\beta})-(\alpha^2+\beta^2)\frac{\partial}{\partial\alpha}(1+e^{\alpha\beta})}{(1+e^{\alpha\beta})^2}=$$

$$=\frac{2\alpha(1+e^{\alpha\beta})-\beta e^{\alpha\beta}(\alpha^2+\beta^2)}{(1+e^{\alpha\beta})^2}=\frac{2\alpha}{1+e^{\alpha\beta}}-\frac{\beta(\alpha^2+\beta^2)e^{\alpha\beta}}{(1+e^{\alpha\beta})^2}$$

Answer: $\frac{\partial}{\partial\alpha}\frac{\alpha^2+\beta^2}{1+e^{\alpha\beta}}=\frac{2\alpha}{1+e^{\alpha\beta}}-\frac{\beta(\alpha^2+\beta^2)e^{\alpha\beta}}{(1+e^{\alpha\beta})^2}$