Question: I am going to play a card game. I get 5 chances to draw an Ace from a 52 card deck. Each time I draw, I do not place the drawn card back into the deck. The game is over when either I draw an Ace, or I have drawn 5 cards without drawing an Ace. It costs me $1 to play. If I draw the Ace of spades, I win $50, if I draw any other Ace I win $10. What is the probability I will draw an Ace (other than spades) and the probability I will draw the Ace of spades.

Solution:

By classical definition of probability let's find probabilities $P(A_1)$, $P(A_2)$ of events A_1, A_2 :

A_1="Draw an A♠ at the 1st turn", $P(A_1)=\frac{1}{52}$.

A_2="Draw an A♠ at the 2nd turn"="draw no ace at the first turn and then draw A♠ at the second one", $P(A_2)=\frac{48}{52}\cdot\frac{1}{51}=\frac{4}{221}$.

By analogy,

A_3="Draw an A♠ at the 3rd turn", $P(A_3)=\frac{48}{52}\cdot\frac{47}{51}\cdot\frac{1}{50}=\frac{94}{5525}$.

A_4="Draw an A♠ at the 4th turn", $P(A_4)=\frac{48}{52}\cdot\frac{47}{51}\cdot\frac{46}{50}\cdot\frac{1}{49}=\frac{4324}{270725}$.

A_5="Draw an A♠ at the 5th turn", $P(A_5)=\frac{48}{52}\cdot\frac{47}{51}\cdot\frac{46}{50}\cdot\frac{45}{49}\cdot\frac{1}{48}=\frac{3243}{216580}$.

Since A="Draw an A♠"=$A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5$

is the union of mutually exclusive events A_1, A_2 ,A_3 ,A_4, A_5

then

$P(A)=P(A_1)+P(A_2)+P(A_3)+P(A_4)+P(A_5)=\frac{1}{52}+\frac{4}{221}+\frac{94}{5525}+\frac{4324}{270725}+\frac{3243}{216580}=\frac{4618}{54145}\approx 0.09.$

Let B be the event "draw no ace". Its probability is

$P(B)=\frac{48}{52}\cdot\frac{47}{51}\cdot\frac{46}{50}\cdot\frac{45}{49}\cdot\frac{44}{48}=\frac{35673}{54145}$

Consider event C="Draw an ace, but not A♠"="Draw any ace" \ "Draw A♠" = (U \B)\A.

Here U is the sample space. Therefore, according to additivity of probability of mutually exclusive events we have

$P(C)=1-P(B)-P(A)=1-\frac{35673}{54145}-\frac{4618}{54145}=\frac{13854}{54145}\approx 0.26.$

Answer: P("Draw an ace, but not A♠")= $\frac{13854}{54145}\approx 0.26;$

P("Draw A♠")= $\frac{4618}{54145}\approx 0.09.$