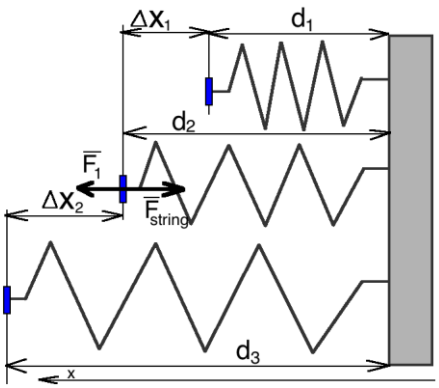
The spring-flex exercise system consists of a spring with one end fixed and a handle on the other end. The idea is that you exercise your muscles by stretching the spring from its natural length, which is 39 cm. If a 190 Newton force is required to keep the spring stretched to a length of 57 cm, how much work is required to stretch it from 57 cm to 64 cm?

Solution:



$d_1 = 0.39\text{m}$ – natural length of the string;
$d_2 = 0.57\text{m}$ –first extension of the spring;
$d_3 = 0.64\text{m}$ –second extension of the spring;
A – work, that is required to stretch string from 57 cm to 64 cm

To find how much work is required to stretch it from 57 cm to 64 cm, first we need to find the spring constant k. Newton's second law for a moment when the spring is stretched on length Δx_1:

$\vec{F}_1 = \vec{F}_{string}$

x: $F_1 - F_{string} = 0$

$F_1 = F_{string}$ (1)

Hooke's law

$F_{string} = k\Delta x_1$

$\Delta x_1 = d_2 - d_1$

$F_{string} = k(d_2 - d_1)$

$$k = \frac{F_{string}}{\Delta x_1} = \frac{F_{string}}{d_2 - d_1} \quad (2)$$

$(1)\text{in}(2)$:

$$k = \frac{F_1}{\Delta x_1} = \frac{F_1}{d_2 - d_1} \quad (3)$$

Mechanical energy when the spring is stretched on length Δx_1:

$$E_{m1} = E_{spring1} = \frac{k\Delta x_1^2}{2} \quad (4)$$

$E_{spring1}$ – the potential energy of the spring

Mechanical energy when the spring is stretched on length $\Delta x_2 + \Delta x_1$:

$$E_{m2} = E_{spring2} = \frac{k\Delta x_2^2}{2} \quad (5)$$

$\Delta x_2 + \Delta x_1 = d_3 - d_1$

$E_{spring2}$ – the potential energy of the spring

The conservation of the total mechanical energy (A - work to increase the potential energy of the spring):

$$E_{m1} + A = E_{m2}$$

$$A = E_{m2} - E_{m1} \qquad (6)$$

$(4)\text{and}(5)\text{in}(6)$:

$$A = \frac{k\Delta x_2^2}{2} - \frac{k\Delta x_1^2}{2} = \frac{k}{2}(d_3 - d_1)^2 - \frac{k}{2}(d_2 - d_1)^2 =$$

$$= \frac{k}{2}((d_3 - d_1)^2 - (d_2 - d_1)^2) \qquad (7)$$

$(2)\text{in}(7)$:

$$A = \frac{F_1((d_3 - d_1)^2 - (d_2 - d_1)^2)}{2 \cdot (d_2 - d_1)} =$$

$$= \frac{190\text{N}((0.64\text{m} - 0.39\text{m})^2 - (0.57\text{m} - 0.39\text{m})^2)}{2(0.57\text{m} - 0.39\text{m})} = 15.9\text{J}$$

Answer: to stretch string from 57 cm to 64 cm we need to do the 15.9J work.