If
$\sin A + \sin^2 A + \sin^3 A = 1$
prove that
$\cos^6 A - 4\cos^4 A + 8\cos^2 A = 4.$

Solution:

We'll use the following trigonometric identity

$\sin^2 A = 1 - \cos^2 A.$

Thus we have

$\sin A + \sin^2 A + \sin^3 A = 1,$
$\sin A + 1 - \cos^2 A + \sin^3 A = 1,$
$\sin A + \sin^3 A - \cos^2 A = 0,$
$\sin A\,(1 + \sin^2 A) = \cos^2 A,$
$\sin A\,(2 - \cos^2 A) = \cos^2 A,$
$\left(\sin A\,(2 - \cos^2 A)\right)^2 = (\cos^2 A)^2,$
$\sin^2 A\,(2 - \cos^2 A)^2 = \cos^4 A,$
$(1 - \cos^2 A)(4 - 4\cos^2 A + \cos^4 A) = \cos^4 A,$

$4 - 4\cos^2 A + \cos^4 A - 4\cos^2 A + 4\cos^4 A - \cos^6 A = \cos^4 A,$

$4 = 8\cos^2 A - 4\cos^4 A + \cos^6 A,$

$\boxed{\cos^6 A - 4\cos^4 A + 8\cos^2 A = 4}$