

A man walks a certain distance. Had he walked half a km an hour faster than he did he would have taken half an hour less, but had he walked half a km an hour slower he would have taken 40 minutes more. Find the distance and his rate of walking.

Solution:

Firstly we express velocity and time of movement. Let v - the velocity the man walked ($\frac{km}{hr}$). Then we can write:

$$\left(v + \frac{1}{2}\right) = \frac{1}{2} \text{ hour less velocity (he walked half a km an hour and taken half an hour less)}$$

$$\left(v - \frac{1}{2}\right) = \frac{40}{60} \text{ hour more velocity (he walked half a km an hour slower and taken 40 minutes more).}$$

Now we express time of movement:

Let t – the time that man walked (hour)

$$\left(t - \frac{1}{2}\right) - \text{time of walking with faster velocity;}$$

$$\left(t + \frac{2}{3}\right) - \text{time of walking with slower velocity.}$$

We can express the time and distance with regard to the expression of speed and time of movement. Typically distance expressed as the product of the time by speed ($D = vt$). First, we express the distance if there is rapid movement:

$$vt = \left(v + \frac{1}{2}\right)\left(t - \frac{1}{2}\right)$$

Simplify the expression.

$$vt = vt - \frac{1}{2}v + \frac{1}{2}t - \frac{1}{4}$$

Thus we get the equation:

$$-\frac{1}{2}v + \frac{1}{2}t = \frac{1}{4}$$

Now we express the distance, given the slow movement:

$$vt = \left(v - \frac{1}{2}\right)\left(t + \frac{2}{3}\right)$$

Simplify the expression.

$$vt = vt + \frac{2}{3}v - \frac{1}{2}t - \frac{2}{6}$$

Thus we get the second equation:

$$\frac{2}{3}v - \frac{1}{2}t = \frac{1}{3}$$

We obtain a system of two equations:

$$\begin{cases} -\frac{1}{2}v + \frac{1}{2}t = \frac{1}{4} \\ \frac{2}{3}v - \frac{1}{2}t = \frac{1}{3} \end{cases}$$

By adding the first equation and the second we get:

$$-\frac{1}{2}v + \frac{1}{2}t + \frac{2}{3}v - \frac{1}{2}t = \frac{1}{4} + \frac{1}{3}$$

Simplify the expression:

$$-\frac{1}{2}v + \frac{1}{2}t + \frac{2}{3}v - \frac{1}{2}t = \frac{1}{4} + \frac{1}{3}$$

$$\frac{1}{6}v = \frac{7}{12}$$

$$v = \frac{7}{2} = 3,5 \text{ km/hr}$$

Thus we have movement velocity, now we can find time of man's movement:

Substitute the found value of velocity in the equation:

$$-\frac{1}{2} \cdot \frac{7}{2} + \frac{1}{2}t = \frac{1}{4}$$

$$-\frac{7}{4} + \frac{1}{2}t = \frac{1}{4}$$

Now we can find the value of time:

$$\frac{1}{2}t = 2 \Rightarrow t = 4 \text{ (hour)}$$

We can also check the speed substituting into the second equation:

$$\frac{2}{3} \cdot \frac{7}{2} - \frac{1}{2}t = \frac{1}{3}$$

$$\frac{14}{6} - \frac{1}{2}t = \frac{1}{3}$$

$$\frac{1}{2}t = 2 \Rightarrow t = 4 \text{ (hour)}$$

On the basis of the obtained values we find the distance:

$$D = vt = 3,5 \times 4 = 14 \text{ (km)}$$

Answer: Distance is 14 *km*, rate of walking is 3,5 *km/hr*.