Answer on question #35321 – Math – Number Theory

James brought fewer donuts than two dozen of donuts. He wants to evenly share them among 2, 3, 4 people. No matter how many people he shares them with he always have one left over. How many did he buy? What numbers goes in to 24 when factored with 2, 3, 4 gives remainder of 1

Answer

Let x is a number of donuts. We know that $x < 24$.

We have the following system of congruence relations

$$\begin{cases} x \equiv 1 \ (mod\ 2) \\ x \equiv 1 \ (mod\ 3) \\ x \equiv 1 \ (mod\ 4) \end{cases}$$

From the first congruence relation we get

$$x = 2t + 1$$

Substitute this into the second congruence relation

$$2t + 1 \equiv 1 \ (mod\ 3) \ \Rightarrow \ 2t \equiv 0 \ (mod\ 3) \ \Rightarrow \ t \equiv 0 \ (mod\ 3)$$

$$t = 3t_1 \ \Rightarrow \ x = 6t_1 + 1$$

Substitute it into the third equation

$$6t_1 + 1 \equiv 1 \ (mod\ 4) \ \Rightarrow \ 6t_1 \equiv 0 \ (mod\ 4) \ \Rightarrow \ t_1 \equiv 0 \ (mod\ 2)$$

$$t_1 = 2t_2 \ \Rightarrow \ x = 12t_2 + 1,$$

where t_2 is integer. Therefore, $x = 13, 25, 37, \ldots$ According to condition $x < 24$ we get that number of donuts is 13.

Answer: 13.