As a balloon rises, its angle of elevation from a point A on level ground 140m. from the point B directly under the balloon changes from 30 degrees to 60 degrees . How far does the balloon rise during this period?

Solution:

$\alpha_1 = 30^o -$ lower angle;

$\alpha_2 = 60^o -$ greater angle;

For the height at lower angle (right triangle ADC) :

$$\tan\alpha_1 = \frac{h_1}{d}$$

$$h_1 = d \cdot \tan\alpha_1 \qquad (1)$$

For the height at greater angle (right triangle BDC) :

$$\tan\alpha_2 = \frac{h_2}{d}$$

$$h_2 = d \cdot \tan\alpha_2 \qquad (2)$$

To find how much balloon was displaced, we need to subtract from the end position (height h_2) the starting position (height h_1):

$$\Delta h = h_2 - h_1 \qquad (3)$$

Substitute (2) and (1) in(3):

$$\Delta h = h_2 - h_1 = d \cdot \tan\alpha_2 - d \cdot \tan\alpha_1 = d(\tan\alpha_2 - \tan\alpha_1)$$
$$= 140\text{m} \cdot (\tan 60^o - \tan 30^o) = 161.7\text{m}$$

Answer: balloon rose during this time on a height of 161.7m