Since W is the spanning set for the subspace S, every vector from the set $V = \{v^1, v^2, \dots, v^M\}$ is expressed as linear combination of vectors from the set W.

$$v^i = c_1^i w^1 + c_2^i w^2 + \cdots + c_N^i w^N$$

Since all the vectors v^i are independent all the vector coefficients

$$c^i = (c_1^i, c_2^i, \dots, c_N^i)$$

are independent.

So we have M linearly independent vectors c^i, each of dimension N.

Suppose $M > N$. Then any N of M vectors c^i form a basis of R^N. Then remaining $M - N$ vectors C_i are expressed via the basis vectors c^i which contradicts to linear independency of c^i. Thus $M \leq N$ and the statement is proved.

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