Since $v^1, \dots, v^m \in S$ and S is the spanning set of vectors $w^1, \dots, w^n$ so each of vectors v^k is a non-trivial linear combination of vectors $w^1, \dots, w^n$. Thus

$$v^k = w^1 c_{1k} + w^2 c_{2k} + \cdots + w^n c_{nk}\,, \qquad k = \overline{1, m}$$

Rewriting this equality we get:

$$v^k = (w^1, w^2, \dots, w^n) \begin{pmatrix} c_{1k} \\ c_{2k} \\ \dots \\ c_{nk} \end{pmatrix}$$

Writing the statements for all $k = \overline{1, m}$ together we get:

$$(v^1, v^2, \dots, v^m) = (w^1, w^2, \dots, w^n) \begin{pmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nm} \end{pmatrix}$$

Using the notations given and denoting

$$C = \begin{pmatrix} c_{11} & \cdots & c_{1m} \\ \vdots & \ddots & \vdots \\ c_{n1} & \cdots & c_{nm} \end{pmatrix}$$

we get:

$$A = BC$$