Suppose L is invertible and lower triangular matrix. We need to prove that L^{-1} is also invertible and low triangular.

Since L is invertible there exists inverse matrix L^{-1} satisfying

$$LL^{-1} = L^{-1}L = I$$

where I is identity matrix. Changing L into L^{-1} in the equalities we get:

$$L^{-1}(L^{-1})^{-1} = (L^{-1})^{-1}L^{-1} = I$$

The following property holds for an invertible matrix L : $(L^{-1})^{-1} = L$. Thus the inverse matrix to L^{-1} exists, so L^{-1} is invertible matrix.

Let's prove that L^{-1} is lower triangular matrix.

Suppose $L^{-1} = [y_1 y_2 \dots y_n]$, so y_i is the $i - th$ column of the matrix L^{-1}.

Then, by definition,

$$LL^{-1} = I$$

$$LL^{-1} = L[y_1 y_2 \dots y_n] = [Ly_1 \dots Ly_n] = [e_1 e_2 \dots e_n]$$

So

$$Ly_k = e_k, k = \overline{1,n}$$

Denoting elements of the matrix L by l_{ij} and elements of the vector y_j by y_j^i we get:

$$l_{11} y_k^1 = 0$$

$$l_{21} y_k^1 + l_{22} y_k^2 = 0$$

...

$$l_{k-1\,1} y_k^1 + \dots + l_{k-1\,k-1} y_k^{k-1} = 0$$

$$l_{k1} y_k^1 + \dots + l_{k\,k} y_k^k = 1$$

Since L is lower triangular matrix then $l_{11} \neq 0$.

The first equation implies $y_k^1 = 0$, then we obtain that the second one implies

$y_k^2 = 0, \dots$, the k-1-th equation implies $y_k^{k-1} = 0$. Thus all the elements of the matrix L^{-1} that are located above the main diagonal are equal to 0, so L^{-1} is lower triangular matrix.