We need to find minimal and maximal value of $2x + y$ on the circle $x^2 + y^2 = 1$.

To do this let's consider Lagrange function:

$$L(x, y, \lambda) = 2x + y - \lambda(x^2 + y^2 - 1)$$

Let's find stationary point:

$$\frac{\partial L}{\partial x} = 2 - 2x\lambda = 0$$

$$\frac{\partial L}{\partial y} = 1 - 2y\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = -(x^2 + y^2 - 1) = 0$$

Solving this we get:

$$x = \frac{1}{\lambda}$$

$$y = \frac{1}{2\lambda}$$

Substituting this to the third equation we get:

$$x^2 + y^2 = \frac{1}{\lambda^2} + \frac{1}{4\lambda^2} = \frac{5}{4\lambda^2} = 1$$

$$\lambda = \pm\frac{\sqrt{5}}{2}$$

Thus extrema points are

$$(x_1, y_1) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

$$(x_2, y_2) = \left(-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$$

$$f(x_1, y_1) = \sqrt{5}$$

$$f(x_2, y_2) = -\sqrt{5}$$

Since circle is a compact the function reaches its maximum and minimum on it. So (x_1, y_1) is point of maximum and (x_2, y_2) is a point of minimum.

Now let's formulate the problem in terms of vectors.

Denote by $v_0 = (2,1)$ and $v_1 = (x, y)$

Then our problem is

$$(v_0, v_1) \to max$$

$$\|v_1\| = 1$$

Since (v_0, v_1) is the linear operator acting on v_1, it's maximizing on the unit sphere is equivalent to find the biggest eigenvalue of this operator, then the corresponding eigenvector will be equal to the direction of the optimal vector.