

Find an approximate value of $\ln(0.9)$ up to 3 decimal places using Maclaurin's expansion.

Solution:

We know

$$\ln(1-x) = -\sum_{n=1}^{\infty} \frac{x^n}{n}, \quad |x| < 1.$$

Thus

$$\ln(1-x) \approx -\sum_{n=1}^N \frac{x^n}{n}.$$

In our case $x = 0.1$. We have to find N . We'll use next condition

$$\frac{x^N}{N} \leq 0.001$$

or

$$\frac{0.1^N}{N} \leq 0.001.$$

If $N = 3$ then

$$\frac{0.1^3}{3} = \frac{1}{3} \cdot 0.001 < 0.001.$$

So we have

$$\ln(0.9) \approx -\frac{0.1}{1} - \frac{0.1^2}{2} - \frac{0.1^3}{3} = -0.105(3)$$

Answer:

$$\ln(0.9) \approx -0.105(3)$$