We need to prove

$$inf x\left(\inf(y) f(x,y)\right) = inf y\left(\inf(x) f(x,y)\right)$$

Let's prove a lemma:

$$\inf_x\left(\inf(y) f(x,y)\right) = \inf_{x,y} f(x,y)$$

Suppose $\inf_{x,y} f(x,y) = \alpha$ (here α may be $-\infty$). Then there exists a sequence of pairs (x_n, y_n) such that

$$\lim_{n\to\infty} f(x_n, y_n) = \alpha$$

Such inequalities hold:

$$\inf(y) f(x_n, y) \le f(x_n, y_n)$$

Thus

$$\inf_x \inf(y) f(x,y) \le \inf_n \inf(y) f(x_n, y) \le \inf_n f(x_n, y_n) = \alpha$$

On the other hand,

$$\forall x \in R\ \ \inf(y) f(x,y) \ge \inf_{x,y} f(x,y) = \alpha$$

Thus,

$$\inf_x \inf(y) f(x,y) \ge \alpha$$

Taking two obtained inequalities together we get:

$$\inf_x \inf(y) f(x,y) = \alpha$$

and the lemma is proved.

Interchanging variables x and y we get such statement:

$$\inf_y \inf(x) f(x,y) = \alpha$$

Taking all together we have:

$$\inf_x \inf(y) f(x,y) = \inf_{x,y} f(x,y) = \inf_y \inf(x) f(x,y)$$

Thus

$$\inf_x \inf(y) f(x,y) = \inf_y \inf(x) f(x,y)$$