

$$f(x) = x^2 + \frac{1}{x^2} + 4x + \frac{4}{x}$$

To find local minima of the function let's make some transformations:

$$f(x) = x^2 + \frac{1}{x^2} + 4x + \frac{4}{x} = \left(x + \frac{1}{x}\right)^2 + 4\left(x + \frac{1}{x}\right) - 2$$

Consider a new variable $y = x + \frac{1}{x}$. Then

$$f(y) = y^2 + 4y - 2$$

This is quadratic function that reaches minimum at $y = -2$. The value of the function equals to $(-2)^2 + 4(-2) - 2 = -6$

To find corresponding x let's solve the equation:

$$x + \frac{1}{x} = -2$$

$$x^2 + 2x + 1 = 0$$

$$(x + 1)^2 = 0$$

$$x = -1$$

Thus the function reaches minimum at point $x = -1$, minimal value equals to -6 .