What is the domain of definition of log(x-1) to the base tan x ?

Domain for log function is:

$y = \log_a x$, where $a > 0$, $a \neq 1$, $x > 0$,

We have:

$$\log_{\tan x}(x - 1)$$

Therefore, $a = \tan x$, $b = x - 1$:

$$\tan x > 0, \tan x \neq 1$$

$$x - 1 > 0 \text{ or } x > 1$$

For $\tan x$ we have two conditions:

$\tan x > 0 \quad \rightarrow \quad 0 + k\pi < x < \frac{\pi}{2} + k\pi, \quad k \in Z;$

$\tan x \neq 1 \quad \rightarrow \quad x \neq \frac{\pi}{4} + k\pi, k \in Z.$

Or, it can be written mathematically:

$x \in \cup_{k \geq 1} \left(k\pi; \frac{\pi}{2} + k\pi\right) \cup \left(1; \frac{\pi}{2}\right) \setminus \left\{\cup_{k \geq 1} \left\{\frac{\pi}{4} + k\pi\right\}\right\}, k \geq 1, k \in Z$

Answer:

$\cup_{k \geq 1} \left(k\pi; \frac{\pi}{2} + k\pi\right) \cup \left(1; \frac{\pi}{2}\right) \setminus \left\{\cup_{k \geq 1} \left\{\frac{\pi}{4} + k\pi\right\}\right\}, k \geq 1, k \in Z.$