

**Answer on question #34945 – Math – Linear Algebra**

can u please help me solve this equation. i have the answer in my book but can u help me solve it

i have a 3x3 matrix

$$1 = p(1,1) + p(2,1)e^{ix} + p(3,1)e^{-ix}$$

$$\cos x = p(2,1) + p(2,2)e^{ix} + p(3,2)e^{-ix}$$

$$\sin x = p(1,3) + p(2,3)e^{ix} + p(3,3)e^{-ix}$$

where p is a 3 by 3 matrix

**Answer:**

As we know

$$e^{ix} = \cos x + i \sin x$$

$$e^{-ix} = \cos x - i \sin x$$

So we get

$$\begin{aligned} 1 &= p(1,1) + p(2,1)e^{ix} + p(3,1)e^{-ix} = \\ &= p(1,1) + (p(2,1) + p(3,1))\cos x + i(p(2,1) - p(3,1))\sin x \end{aligned}$$

It holds when  $p(1,1) = 1, p(2,1) = p(3,1) = 0$ .

$$\begin{aligned} \cos x &= p(2,1) + p(2,2)e^{ix} + p(3,2)e^{-ix} = \\ &= p(2,1) + (p(2,2) + p(3,2))\cos x + i(p(2,2) - p(3,2))\sin x \end{aligned}$$

It holds when  $p(2,1) = 0, p(2,2) = p(3,2) = 0.5$ .

$$\begin{aligned} \cos x &= p(1,3) + p(2,3)e^{ix} + p(3,3)e^{-ix} = \\ &= p(1,3) + (p(2,3) + p(3,3))\cos x + i(p(2,3) - p(3,3))\sin x \end{aligned}$$

It holds when  $p(2,1) = 0, p(2,2) = \frac{1}{2i}, p(3,2) = -1/2i$ .

Therefore we get

$$p = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.5 & 0.5 \\ 0 & \frac{1}{2i} & -\frac{1}{2i} \end{pmatrix}.$$