An insurance company has discovered that only about 0.1% of the population is involved in a certain type of accident each year. If its 10000 policyholders were randomly selected from the population, what is the probability that not more than 5 of its clients are involved in such an accident next year?

Solution.

Because of the low probability, use a formula for the Poisson distribution:

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda},$$

In our case,

$$\lambda = 10000 \cdot 0.001 = 10$$

So

$$P(0 \le k \le 5) = \sum_{k=0}^{5} \left[\frac{10^k}{k!} e^{-10} \right] = \left(\frac{10^0}{0!} + \frac{10^1}{1!} + \cdots + \frac{10^5}{5!} \right) \cdot e^{-10} = \frac{4433}{3e^{10}} \approx 0.0671$$

Answer: 0.0671.