Answer on question #34678 – Math – Real Analysis

Show that the length of the curve y = log Sec x between the points x = 0 and x = pi/3 is log(2 + √3)

Solution

Using the formula

$$l(f) = \int_a^b \sqrt{1 + \left(f'(x)\right)^2}\,dx$$

We obtain

$$l(y) = \int_0^{\pi/3} \sqrt{1 + ((\ln \operatorname{Sec} x)')^2}\,dx \qquad (*)$$

$$(\ln \operatorname{Sec} x)' = \frac{1}{\sec x}(\sec x)' = \cos x\left(\frac{\sin x}{\cos^2 x}\right) = \tan x$$

Substitute this into (*) we get

$$l(y) = \int_0^{\frac{\pi}{3}} \sqrt{1 + (\tan x)^2}\,dx = \int_0^{\frac{\pi}{3}} \sqrt{\frac{1}{\cos^2 x}}\,dx = \int_0^{\frac{\pi}{3}} \frac{1}{\cos x}\,dx = \int_0^{\frac{\pi}{3}} \sec x\,dx =$$

$$= \ln|\sec x + \tan x||_0^{\frac{\pi}{3}} = \ln(2 + \sqrt{3}).$$

QED.