

Real part of an analytic function is

$$u(x, y) = 3x^2y - y^3 - x + 6$$

To find the imaginary part of the function let's use Cauchy-Riemann equations:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

Calculating derivatives we get:

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -(3x^2 - 3y^2) = 3y^2 - 3x^2$$

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = 6xy - 1$$

Let's find $v(x, y)$.

$$v(x, y) = \int \frac{\partial v}{\partial x} dx + c(y) = 3y^2x - x^3 + c(y)$$

where $c(y)$ is an arbitrary function of y . Let's differentiate the last equality by y :

$$\frac{\partial v}{\partial y} = 6xy + c'(y) = 6xy - 1$$

So $c'(y) = -1$, so $c(y) = -y + c$, c is arbitrary constant. So

$$v(x, y) = 3y^2x - x^3 + c(y) = 3y^2x - x^3 - y + c$$

So analytic function with real part $u(x, y)$ is

$$u(x, y) + iv(x, y) = 3x^2y - y^3 - x + 6 + i(3y^2x - x^3 - y + c)$$