Answer on question #34456 – Math – Analytic Geometry

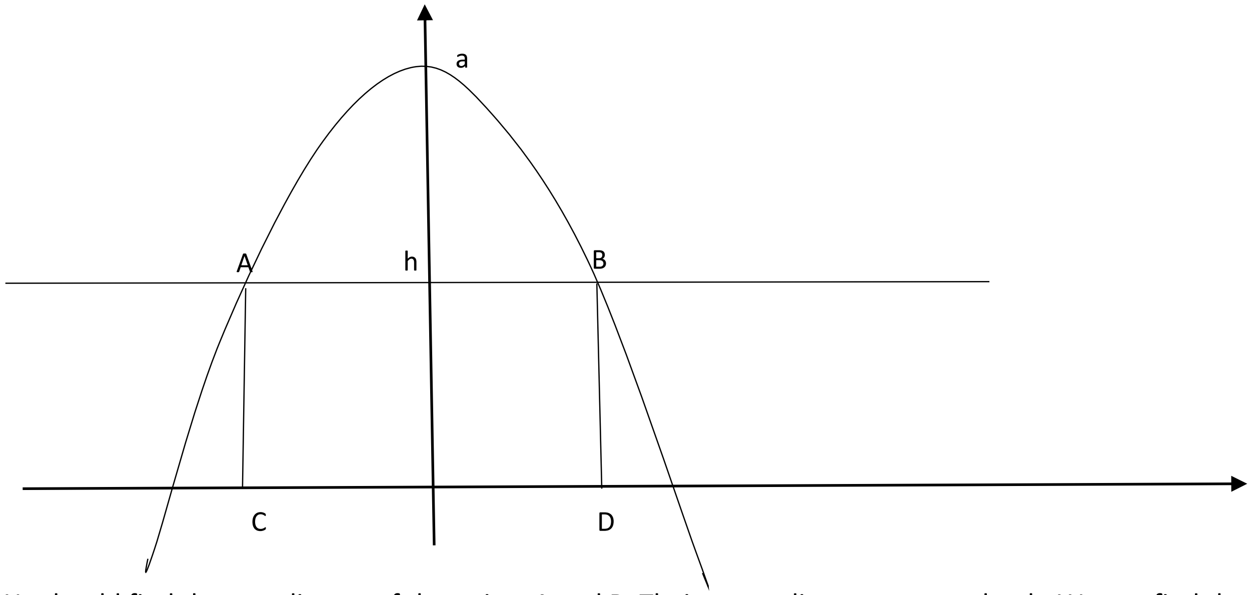
The parabola P: y=a-x^2, a>0, is cut by the line L: y=h, h>0, in the points A and B. the points C and D, on the x-axis, are such that ABCD is a rectangle, R.

(a) Find the coordinates for the vertices of R.

(b) Find the value(s) of h so that R has the greatest area.

Solution

Let us graph this rectangular



We should find the coordinates of the points A and B. Their y-coordinates are equal to h. We can find the x-coordinates from the equation

$$h = a - x^2$$

$$x = \pm\sqrt{a-h}$$

Therefore, we get these two points: $A\left(-\sqrt{a-h}; h\right),\ B\left(\sqrt{a-h}; h\right)$.

Points C and D have zero y-coordinates and the same x-coordinates: $C\left(-\sqrt{a-h}; 0\right),\ B\left(\sqrt{a-h}; 0\right)$.

The area of this rectangular is

$$S = 2h\sqrt{a-h}$$

To find the maximum of this value we should find the derivative of S with respect to h and equate it to zero

$$S' = 2\sqrt{a-h} - \frac{h}{\sqrt{a-h}} = \frac{2a-2h-h}{\sqrt{a-h}} = \frac{(2a-3h)}{\sqrt{a-h}} = 0$$

$$2a - 3h = 0, \qquad \Rightarrow\ h = \frac{2a}{3}.$$

At this point the function S(x) has the maximum.

Answer: (a) $A\left(-\sqrt{a-h}; h\right),\ B\left(\sqrt{a-h}; h\right), C\left(-\sqrt{a-h}; 0\right),\ B\left(\sqrt{a-h}; 0\right)$.

(b) $h = \frac{2a}{3}$.