Show that the length of the curve y=log secx between the points x = 0 and x = π/3 is log(2+√3)

Solution:

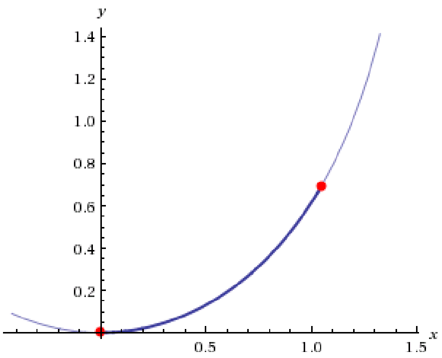
The give curve is $y = \log\sec x$ (1)

Differentiating (1) w.r.t x, we get:

$$\frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \cdot \tan x = \tan x$$

Now $\left(\frac{dS}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$ (2)

If the arc length **S** of the given curve is measured from $x = 0$ in the direction of x increasing, we have:



$$\frac{ds}{dx} = \sec x; ds = \sec x\, dx$$

Therefore if S_1 denotes the arc length from $x = 0$ to $= \frac{1}{3}\pi$, then

$$\int_0^{S_1} ds = \int_0^{\frac{\pi}{3}} \sec x\, dx = [\log(\sec x + \tan x)]_0^{\frac{\pi}{3}}$$

$$S_1 = \left[\log(\sec\frac{\pi}{3} + \tan\frac{\pi}{3}) - \log 1\right] = \log(2+\sqrt{3}) - 0 = \log(2+\sqrt{3})$$

Answer: length of the curve $y = \log\sec x$ between the points x = 0 and x = $\frac{\pi}{3}$ is $\log(2+\sqrt{3})$