

Consider the function $y = x^2 - 2x - 15$ and determine the following:

- Whether the function has a minimum or a maximum
- Minimum or a maximum value of the function
- Crossing points of the graph in x-axis
- The intercept

Solution:

a) $y = x^2 - 2x - 15$ or $f(x) = x^2 - 2x - 15$;

The first step we take the first derivative of a function $f(x)$ and equate it to zero.

$$f'(x) = 2x - 2 \text{ then } 2x - 2 = 0; \text{ and } x = 1;$$

$x = 1$ is the critical value.

$$f(x) = x^2 - 2x - 15; f(1) = 1^2 - 2 \cdot 1 - 15 = -16$$

The extreme value is -16 .

$$f''(x) = 2; f''(x) \text{ evaluated at the critical value } 1.$$

$f''(1) = 2$ and $2 > 0$. It means that the critical value 1 determines a minimum.

b) In point $(1, -16)$ the function has a minimum; -16 is the minimum value of function $f(x)$.

c) x- intercept is the point where a line crosses the x-axis. It crosses the x-axis when $y=0$ or $f(x) = 0$;

$$x^2 - 2x - 15 = 0$$

$$(x+3)(x-5) = 0$$

$$x = -3 \text{ or } x = 5$$

d) y- intercept is the point where a line crosses the y-axis. It crosses the y-axis when $x=0$;

$$y = x^2 - 2x - 15; \text{ when } x=0; \text{ then } y = -15;$$