Task. The movement of the crest of a wave is modeled with the equation $h(t) = 0.2\cos 4t + 0.3\sin 5t$. Find the maximum height of the wave and the time at which it occurs.

Solution. The maximum of the function $h(t)$ is achieved at some critical point, i.e. a point t such that $h'(t) = 0$. So one of the ways to find the maximum of h is to compute all critical point of h and the take the maxiaml value of h amont these points. So we should solve the equation

$$h'(t) = 0.$$

However

$$h'(t) = 0.2 * 4 * (-\sin 4t) + 0.3 * 5 * \cos 5t = -0.8\sin 4t + 1.5\cos 5t$$

and so the equation is hard to solve, since the frequencies at sin and cos are distinct.

Nevertheless we can use another method. Notice that

$$\max_{x\in\mathbb{R}} \cos x = \max_{x\in\mathbb{R}} \sin x = 1.$$

So one of the ways is to find a point $\bar{t}$ such that

$$\cos 4\bar{t} = \sin 5\bar{t} = 1.$$

Then

$$\max h = h(\bar{t}) = 0.2 * 1 + 0.3 * 1 = 0.5.$$

From the equations

$$\cos 4\bar{t} = \sin 5\bar{t} = 1$$

we get

$$4\bar{t} = 2\pi k, \qquad 5\bar{t} = \frac{\pi}{2} + 2\pi l = \frac{(4l+1)\pi}{2},$$

for some $k, l = 0, \pm 1, \pm 2, \ldots$.

Hence

$$\bar{t} = \frac{\pi k}{2} = \frac{(4l+1)\pi}{2 * 5}$$
$$k = \frac{4l+1}{5}$$
$$4l + 1 = 5k.$$

One of the solutions is $l = 1$ and $k = 1$. In this case

$$4l + 1 = 5k = 5.$$

Hence

$$\bar{t} = \frac{2\pi k}{4} = \frac{2\pi * 1}{4} = \frac{\pi}{2}.$$

In this case

$$\cos 4\bar{t} = \cos \frac{4\pi}{2} = \cos 2\pi = 1,$$
$$\sin 5\bar{t} = \sin \frac{5\pi}{2} = 1,$$

and so

$$\max h = h(\bar{t}) = 0.5.$$

Answer. $\max h = h(\frac{\pi}{2}) = 0.5$.