Task. Find the angle (in degrees) between the longest edge and the longest diagonal of a 2 by 5 by 6 rectangular box.

Solution. Choose coordinats (x, y, z) so that one of the vertics of the box has coordinates $O = (0, 0, 0)$, and other vertices adjacent to O are

$$A = (2, 0, 0), \qquad B = (0, 5, 0), \qquad C = (0, 0, 6).$$

Let $D = (2, 5, 6)$ be the verex opposite to O. Then the longest edge is OC, and the longest diagonal is OD.

So we have to find the angle α between the vectors

$$\vec{OC} = (0, 0, 6) \qquad \text{and} \qquad \vec{OD} = (2, 5, 6).$$

Notice that their scalar product is

$$\vec{OC} \cdot \vec{OD} = 0 * 2 + 0 * 5 + 6 * 6 = 36.$$

On the other hand, this their scalar product can be computed as follows:

$$\vec{OC} \cdot \vec{OD} = |OC| \cdot |OD| \cdot \cos\alpha,$$

whence

$$\cos\alpha = \frac{\vec{OC} \cdot \vec{OD}}{|OC| \cdot |OD|}.$$

Since

$$|OC| = \sqrt{0^2 + 0^2 + 6^2} = \sqrt{36} = 6,$$
$$|OD| = \sqrt{2^2 + 5^2 + 6^2} = \sqrt{65},$$

we obtain that

$$\cos\alpha = \frac{\vec{OC} \cdot \vec{OD}}{|OC| \cdot |OD|} = \frac{36}{6 * \sqrt{65}} = \frac{6}{\sqrt{65}} \approx 0.74421.$$

Hence

$$\alpha = \arccos 0.74421 = 41.9^\circ.$$

Answer. 41.9°.