In R, define $a \circ b = a + b - ab$. Show that this binary operation is associative, and that $(R,\circ)$ is a monoid with zero as the identity element.

Solution.

For $\forall\ a, b, c \in \mathbb{R}$, we have

$$(a \circ b) \circ c = (a + b - ab) \circ c = a + b + c - ab - bc - ac + abc = a \circ (b + c - bc) = a \circ (b \circ c)$$

Furthermore, $a \circ 0 = a + 0 - a \cdot 0 = a$ and $0 \circ a = 0 + a - 0 \cdot a = a$, so $(R,\circ)$ is indeed a monoid with zero as the identity element.