Answer on Question #34146 – Math – Algebra

Kindly tell me how to solve (1+0.075)^10

Solution

We use the Binomial Theorem as follows:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k, \qquad (1)$$

where

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$, $n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1) \cdot n$

In this question

$$a = 1,\ b = 0.075,\ n = 10,\ k = 0; 1; 2; 3; 4; 5; 6; 7; 8; 9; 10. \qquad (2)$$

Calculate

$\binom{10}{0} = \frac{10!}{0!10!} = 1$, $\binom{10}{1} = \frac{10!}{1!(10-1)!} = \frac{10!}{9!} = 10$, $\binom{10}{2} = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10\cdot 9}{2} = 5 \cdot 9 = 45$,

$\binom{10}{3} = \frac{10!}{3!(10-3)!} = \frac{10!}{3!7!} = \frac{10\cdot 9\cdot 8}{3\cdot 2} = \frac{720}{6} = 120$, $\binom{10}{4} = \frac{10!}{4!(10-4)!} = \frac{10!}{4!6!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2} = 210$,

$\binom{10}{5} = \frac{10!}{5!(10-5)!} = \frac{10!}{5!5!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6}{5\cdot 4\cdot 3\cdot 2} = 252$.

Apply the property $\binom{n}{m} = \binom{n}{n-m}$ to calculate other binomial coefficients:

$\binom{10}{6} = \binom{10}{10-6} = \binom{10}{4} = 210$, $\binom{10}{7} = \binom{10}{10-7} = \binom{10}{3} = 120$, $\binom{10}{8} = \binom{10}{10-8} = \binom{10}{2} = 45$,
$\binom{10}{9} = \binom{10}{10-9} = \binom{10}{1} = 10$, $\binom{10}{10} = \binom{10}{10-10} = \binom{10}{0} = 1$,

Plug (2) into (1) and obtain

$$(1+0.075)^{10} = \binom{10}{0} 1^{10-0} 0.075^0 + \binom{10}{1} 1^{10-1} 0.075^1 + \binom{10}{2} 1^{10-2} 0.075^2 + \binom{10}{3} 1^{10-3} 0.075^3 +$$
$$+ \binom{10}{4} 1^{10-4} 0.075^4 + \binom{10}{5} 1^{10-5} 0.075^5 + \binom{10}{6} 1^{10-6} 0.075^6 + \binom{10}{7} 1^{10-7} 0.075^7 +$$
$$+ \binom{10}{8} 1^{10-8} 0.075^8 + \binom{10}{9} 1^{10-9} 0.075^9 + \binom{10}{10} 1^{10-10} 0.075^{10} =$$

$$= 1 + 10 \cdot 0.075 + 45 \cdot 0.075^2 + 120 \cdot 0.075^3 + 210 \cdot 0.075^4 + 252 \cdot 0.075^5 + 210 \cdot 0.075^6 +$$
$$+ 120 \cdot 0.075^7 + 45 \cdot 0.075^8 + 10 \cdot 0.075^9 + 0.075^{10} = 2{,}061032$$