Show that the length of the curve y=log sec x between the points x=0 and x=π/3 is log (2+√3).
Solution:

The Length of the curve y=log sec x between the points x=0 and x=π/3 given a formula

$$s = \int_0^{\pi/3} \sqrt{1+(y')^2}$$

$$y' = (\log(\sec(x)))' = \frac{1}{\sec(x)}(\sec(x))' = \tan(x)$$

$$\sqrt{1+tan^2 x} = \frac{1}{\cos(x)}$$

$$s = \int_0^{\pi/3} \frac{dx}{\cos(x)} = log\left|\tan(\frac{x}{2}+\frac{\pi}{4})\right|_0^{\pi/3} = log\left|\tan(\frac{\pi}{6}+\frac{\pi}{4})\right| - log\left|\tan\left(\frac{\pi}{4}\right)\right|$$

$$\tan\left(\frac{\pi}{6}+\frac{\pi}{4}\right) = \frac{\tan\left(\frac{\pi}{6}\right)+\tan(\frac{\pi}{4})}{1-\tan\left(\frac{\pi}{6}\right)*\tan(\frac{\pi}{4})} = \frac{\frac{\sqrt{3}}{3}+1}{1-\frac{\sqrt{3}}{3}} = 2+\sqrt{3}$$

So

$$s = \int_0^{\pi/3} \frac{dx}{\cos(x)} === log\left|\tan(\frac{\pi}{6}+\frac{\pi}{4})\right| - log\left|\tan\left(\frac{\pi}{4}\right)\right| = \log(2+\sqrt{3})$$