1. Show that the length of the curve $y = \log \sec x$ between the points $x = 0$ and $x = \pi$ is $\log\left(2 + \sqrt{3}\right)$.

Solution.

The length of the curve $y = y(x)$ between the points $x = a$ and $x = b$ is calculated by the formula:

$$l = \int_a^b \sqrt{1 + (y')^2}\,dx. \qquad (1)$$

The derivative of the given function (assuming the natural logarithm by notation $\log$):

$$y' = \frac{1}{\sec x} \cdot (\sec x)' = \cos x \cdot \left(\frac{1}{\cos x}\right)' = \cos x \cdot \left(-\frac{1}{\cos^2 x} \cdot\right) \cdot (\cos x)' = -\frac{1}{\cos x} \cdot (-\sin x) = \tan x.$$

So, the length of the given curve ought to be calculated by the following integral:

$$l = \int_0^\pi \sqrt{1 + \tan^2 x}\,dx = \int_0^\pi \frac{dx}{|\cos x|}.$$

Nevertheless, the formula (1) is used for the function, which is finite and continuous at the interval $x \in [a; b]$. But the given function $y = \log \sec x$ is not defined at the interval $x \in \left[\frac{\pi}{2}; \pi\right]$, because $\cos x < 0$ for $x \in \left[\frac{\pi}{2}; \pi\right]$, but all logarithms are defines for positive arguments only.

So, the question is not correct.

Answer: incorrect condition.