$$f(x)=\sum_{k=0}^{+\infty}\frac{f^{(k)}(0)x^k}{k!}$$ -- general formula

$$(\sin x)^{(1)}=\cos x=\sin\left(x+\frac{\pi}{2}\right)$$

$$(\sin x)^{(2)}=-\sin x=\sin(x+\pi)$$

$$(\sin x)^{(3)}=-\cos x=\sin\left(x+\frac{3\pi}{2}\right)$$

$$(\sin x)^{(4)}=\sin x=\sin(x+2\pi)$$

With the help of mathematical induction we can prove that

$$(\sin x)^{(k)}=\sin\left(\frac{\pi}{2}k+x\right)$$

Hence, $(\sin x)^{(k)}(0)=\sin\left(\frac{\pi}{2}k\right)=\begin{cases}(-1)^k, k=4p\pm1\\ 0, k=4p,4p+2\end{cases}$

If we consider only those values of k, for which $(\sin x)^{(k)}(0)\neq 0$ ($k=4p\pm1$), then k will be only odd

Because of the fact that all summands with even indexes equal to zero (because of previous fact), we can consider only odd indexes of summands.

So, the final formula is

$$\sin x=\sum_{k=0}^{+\infty}\frac{(-1)^k x^{2k+1}}{(2k+1)!}$$