

**Answer on question 33904 – Math – Number Theory**

if  $a, b, c$  are any three integers such that  $(a,c)=1$  and  $(b,c)=1$ , then show that  $(ab,c)=1$ .

**Solution**

Let the prime factorizations of  $a, b$  and  $c$  are

$$a = p_1^{a_1} \dots p_k^{a_k}, \quad b = q_1^{b_1} \dots q_n^{b_n}, \quad c = r_1^{c_1} \dots r_m^{c_m}$$

From the condition  $(a,c)=1$  we get that for any  $i = 1 \dots k, j = 1 \dots m, p_i \neq r_j$ .

From the condition  $(b,c)=1$  we get that for any  $i = 1 \dots m, j = 1 \dots n, r_i \neq q_j$ .

Then the prime factorization of  $ab$  is

$$ab = p_1^{a_1} \dots p_k^{a_k} q_1^{b_1} \dots q_n^{b_n}$$

As any of this multipliers doesn't equal to any of the multipliers of  $c$  then  $(ab,c)=1$ .

**QED.**