Task.

Let a, b, c be integers such that $gcd(a, b, c) = 1$. Find $gcd(a + b, b + c, c)$.

Solution. Recall that $gcd(a, b, c) = 1$ if and only if there exist integer numbers x, y, z such that

$$ax + by + cz = 1.$$

We claim that $gcd(a+b, b+c, c) = 1$ as well. For this it siffices to find numbers x', y', z' such that

$$(a + b)x' + (b + c)y' + cz' = 1.$$

From identity

$$ax + by + cz = 1$$

we get

$$\begin{aligned}
1 &= ax + by + cz \\
&= ax + bx - bx + by + cz \\
&= (a + b)x + b(y - x) + cz \\
&= (a + b)x + b(y - x) + c(y - x) - c(y - x) + cz \\
&= (a + b)x + (b + c)(y - x) - c(y - x) + cz \\
&= (a + b) \cdot \underbrace{x}_{x'} + (b + c) \cdot \underbrace{(y - x)}_{y'} + c \cdot \underbrace{(z - y + x)}_{z'}.
\end{aligned}$$

So we can put

$$x' = x, \qquad y' = y - x, \qquad z' = z - y + x.$$

Answer. $gcd(a, b, c) = 1$