Prove that $\gcd(n-1,\ n+1) = 1$ or 2 for each $n \geq 2$ and $\gcd(2n-1,\ 2n+1) = 1$ for each $n \geq 7$.

Solution.

1. Assume that $n-1$ divides by d and $n+1$ divides by d.

Then $(n+1)-(n-1)$ divides by d and $(n+1)-(n-1) = 2$ (the difference of the two).

The positive integer divisors of 2 is 1 or 2, so $\gcd(n-1,\ n+1) = 1$ or 2 for each $n \geq 2$.

2. Assume that $2n-1$ divides by d and $2n+1$ divides by d.

Then $(2n+1)-(2n-1)$ divides by d and $(2n+1)-(2n-1) = 2$.

The positive integer divisors of 2 is 1 or 2, but $n \geq 7$, so $\gcd(2n-1,\ 2n+1) = 1$ for each $n \geq 7$.