

Using the definition,

$$\gcd(b, c) = d$$

such that  $d|a; d|b$  and if  $f|a, f|b$  then  $f|d$

So

$$\gcd(a, \gcd(b, c)) = \gcd(a, d) = e$$

Again, using the definition,  $e|a$  and  $e|d$ . Since  $d|a$  and  $d|b$  we get  $e|a$  and  $e|b$ . So  $e|a; e|b; e|c$ .

Next, if  $f|a$  and  $f|d$  then  $f|e$ . Using the fact  $d|b$  and  $d|c$  then we have implication

$$f|a, f|b, f|c \Rightarrow f|e$$

So

$$\gcd(a, \gcd(b, c)) = \gcd(a, (b, c)) = \gcd(a, b, c)$$

Using the symmetry

$$\gcd(a, b, c) = \gcd(a, (b, c)) = \gcd((a, b), c) = \gcd((a, c), b)$$