

**Task.** Given that  $A \times B = O$ ,  $B \times C = O$ , and  $A \neq 0$ ,  $B \neq 0$ ,  $C \neq 0$ . Find the value of  $A \times C$ .

**Solution.** Recall that for any two vectors  $A$  and  $B$  in  $\mathbb{R}^3$  the absolute value of their cross product  $|A \times B|$  is equal to

$$|A \times B| = |A| \cdot |B| \cdot \sin \widehat{AB},$$

where  $\widehat{AB}$  is the angle between vectors  $A$  and  $B$ .

By assumption  $|A| \neq 0$ ,  $|B| \neq 0$ ,  $|C| \neq 0$ , and

$$|A| \cdot |B| \cdot \sin \widehat{AB} = 0, \quad |B| \cdot |C| \cdot \sin \widehat{CB} = 0.$$

Therefore

$$\sin \widehat{AB} = \sin \widehat{CB} = 0.$$

This means that vectors  $A$  and  $B$  are collinear, and similarly  $B$  and  $C$  are also collinear. Hence,  $A$  and  $C$  are also collinear, and so

$$\sin \widehat{AC} = 0.$$

Therefore

$$|A \times C| = |A| \cdot |C| \cdot \sin \widehat{AC} = 0,$$

which means that  $A \times C = 0$ .

**Answer.**  $A \times C = 0$ .